

# A physics regularized Machine Learning Model for traffic state estimation

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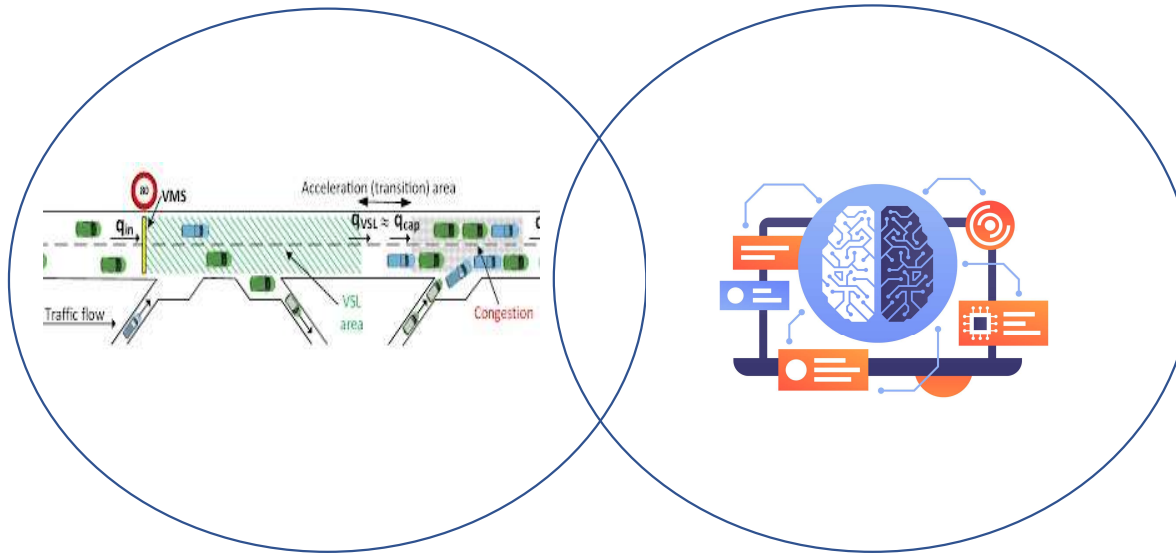
# Outline

- Research proposal and Motivation
- Traffic model used
- ML algorithm used
- Hybrid model
- Algorithm
- Case study
- Results



# Research proposal

CREATE A HYBRID MODEL



classical traffic models

Machine Learning models



# Motivation

## Classical models

strong assumptions

require effort in  
parameter calibrations

fall short of capturing  
data uncertainties

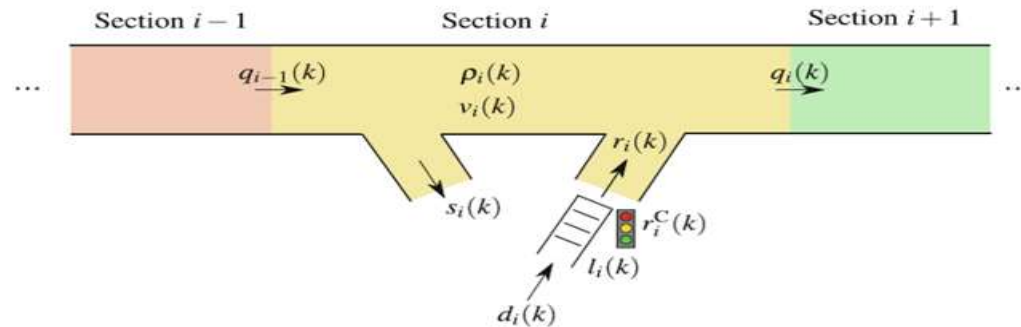
## ML models

highly depend on the data  
quality

results are usually hard to  
be interpreted

Get the best of both worlds!

# METANET



$$\rho_i(k+1) = \rho_i(k) + \frac{T}{L_i \lambda_i} [q_{i-1}(k) - q_i(k) + r_i(k) - s_i(k)]$$

$$v_i(k+1) = v_i(k) + \frac{T}{\tau} [V_i(k) - v_i(k)] + \frac{T}{L_i} v_i(k) [v_{i-1}(k) - v_i(k)] - \frac{vT[\bar{\rho}_{i+1}(k) - \bar{\rho}_i(k)]}{\tau L_i [\bar{\rho}_i(k) + \chi]} - \Delta T \frac{v_i(k) r_i(k)}{L_i [\bar{\rho}_i(k) + \chi]}$$

$$q_i(k) = \rho_i(k) \cdot v_i(k) \cdot \lambda_i$$

$\tau, \chi, \Delta, v$  - model parameters

$v_i^f$  - free flow speed  
 $\rho_i^{max}$  - jam density.

**Steady state speed-density relation:**

$$V_i(k) = V(\bar{\rho}_i(k)) = v_i^f \cdot [1 - (\frac{\bar{\rho}_i(k)}{\rho_i^{max}})^l]^m$$



# Gaussian Processes (GP)

- Framework for non-parametric regression
- Model the data points as jointly Gaussian

$$y_1, \dots, y_n \mid x_1, \dots, x_n \sim N(\mu, \Sigma)$$

- Predictive model for an input trajectory

$$p(f(x^*) \mid x^*, X, Y) = N(\mu(x^*), \sigma(x^*))$$

provides a mean and a predictive variance:

$$\mu(x^*) = K_*^T (K + \tau^{-1} I)^{-1} Y$$

$$\sigma(x^*) = K(x^*, x^*) - K_*^T (K + \tau^{-1} I)^{-1} K_*$$

where  $K_* = [K(x^*, x_1) \dots K(x^*, x_n)]^T$  is the kernel

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# Hybrid model

Use METANET equations in the regularization process of ML algorithm



Latent Force Functions – to  
encode physics into GP

Evidence Lower Bound  
(ELBO) of a posterior  
distribution

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# Objective function

- Derive the objective function from posterior probability

$$\max L = \sum_{i=1}^{d'} \log[N([Y]_i | [\mu_f]_i, [\sigma_f]_i)] + \sum_{w=1}^W \gamma_w E_{p(Z)} E_{p(\mu_{f_w} | Z, X, Y)} [\log [N(\Phi_{\mu_{f_w}} | \mu_{g_w}, \sigma_{g_w})]]$$

$$\sigma_f = K_f(X, X) + \tau^{-1} I$$

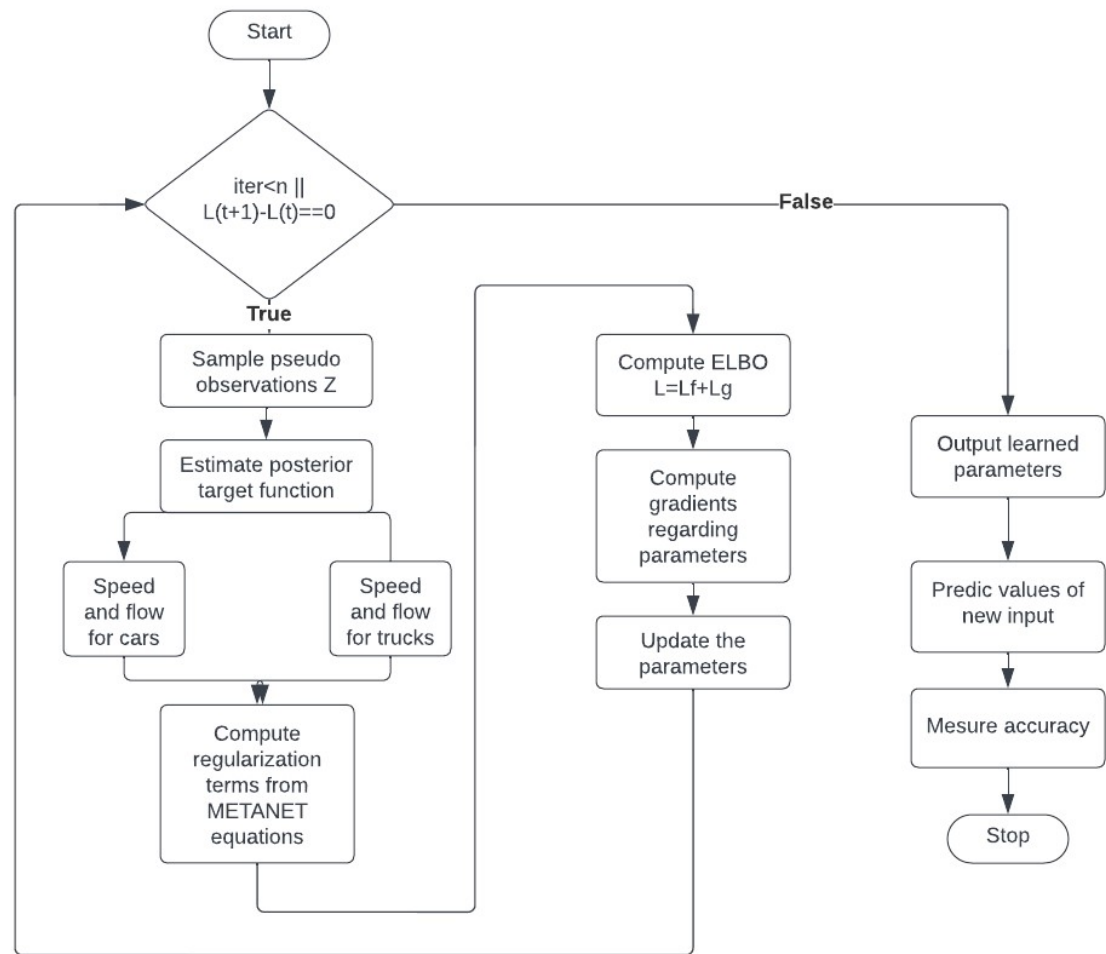
$$\sigma_g = K_g(Z, Z)$$

$$\Theta = [\theta_f \ \theta_g]^T = [\bar{\tau} \ \eta \ \tau \ \nu \ \delta \ \kappa \ \nu_f \ \rho_{cr} \ \alpha]^T$$

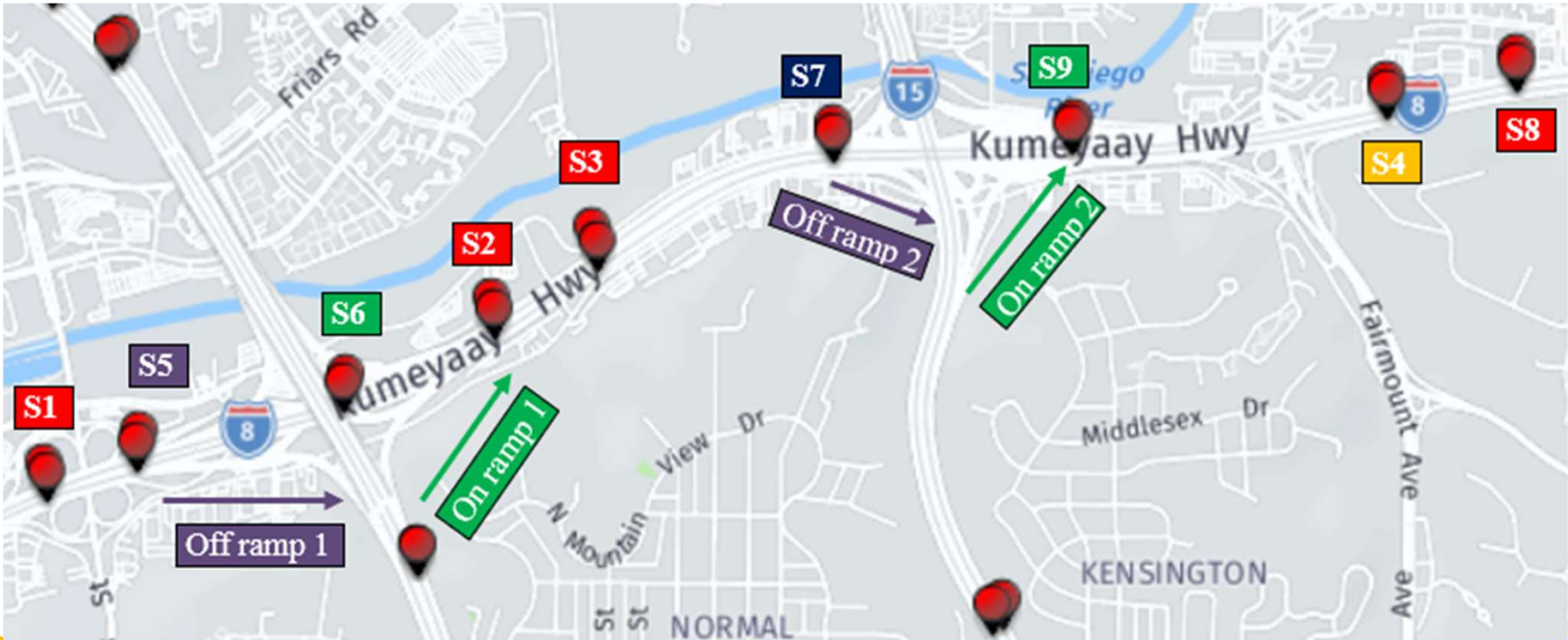
- Objective function has two parts:
    - I. posterior probability where input data are the observed ones
    - II. posterior probability where input data are the pseudo-observations
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# Algorithm

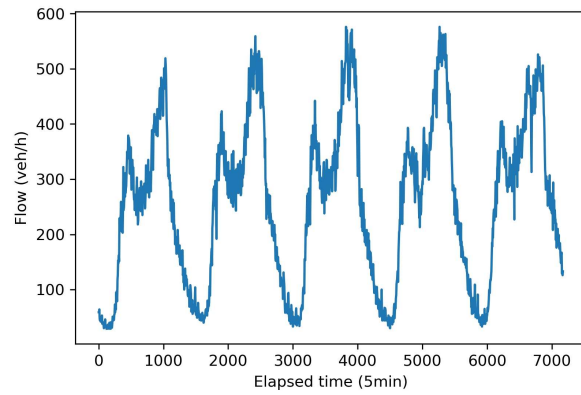


# Case study

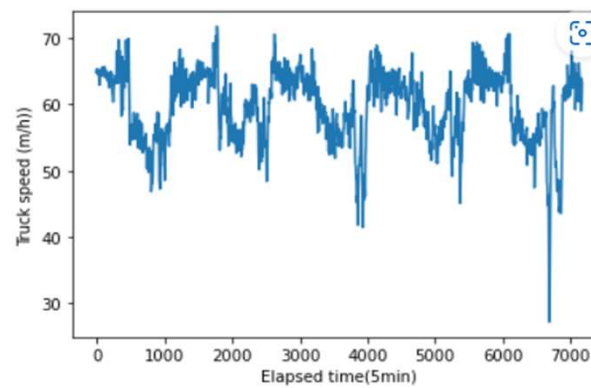
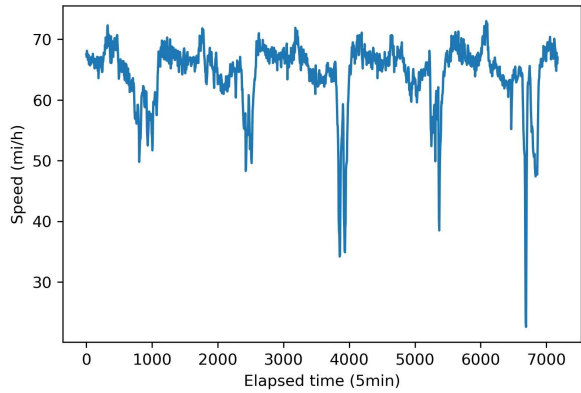
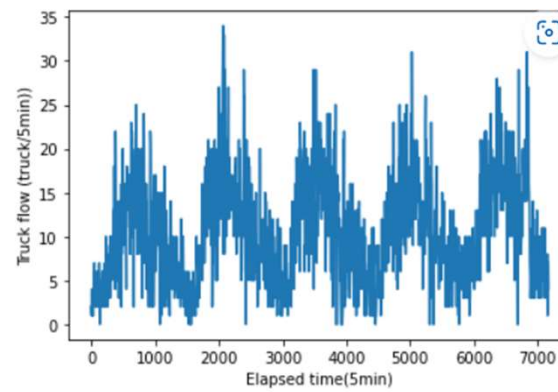


# Input data (two-class case)

Passenger vehicles



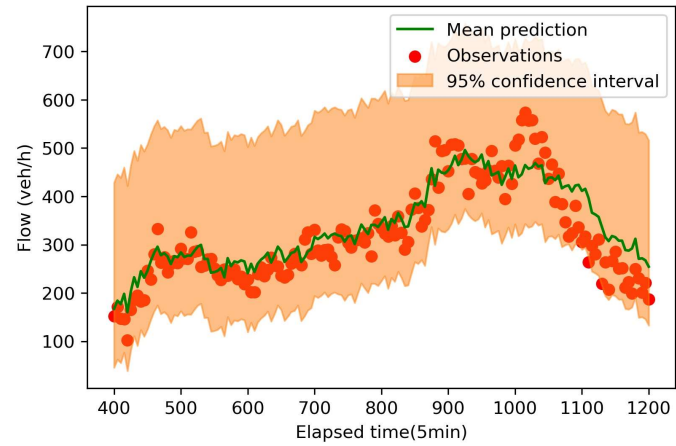
Heavy-duty vehicles



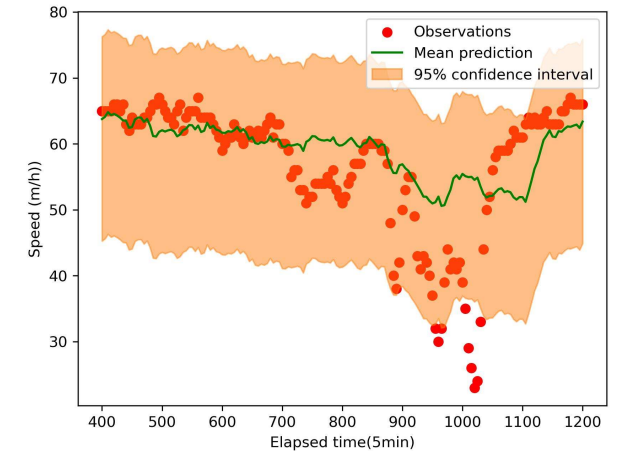
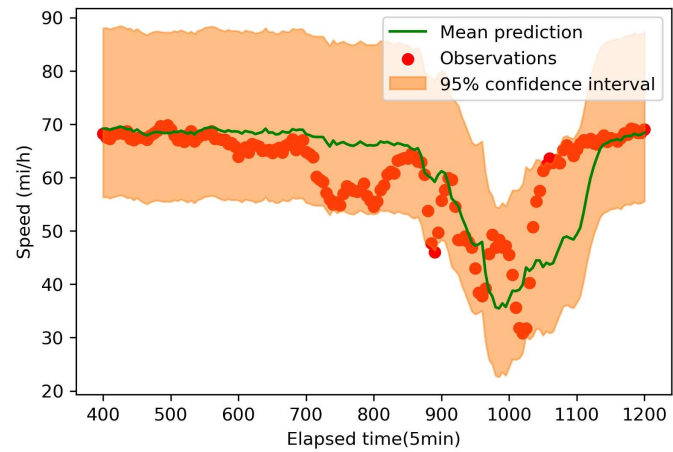
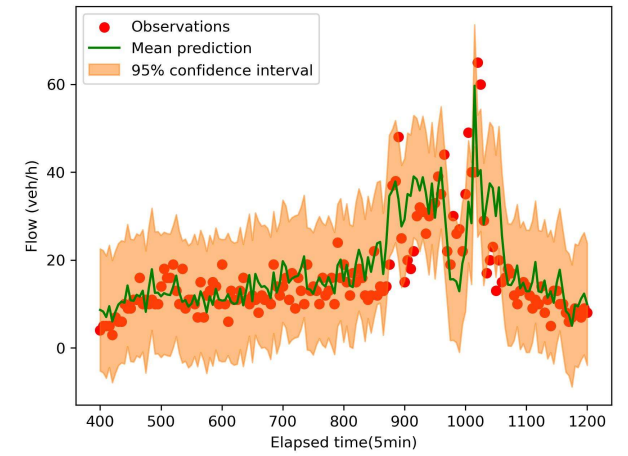
# Results



### Passenger vehicles



### Heavy-duty vehicles





## Results comparison

Method	MAPE flow (passenger vehicles)	MAPE speed (passenger vehicles)	MAPE flow (heavy-duty vehicles)	MAPE speed (heavy-duty vehicles)
Pure ML	0.265	0.115	0.5	0.124
Pure multi-class METANET	0.173	0.13	0.45	0.11
Hybrid model	0.128	0.08	0.27	0.07

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# Conclusions

- We proposed the development of a Machine Learning component capable of approximating a multi-class METANET model
- We can use this hybrid model to predict the traffic state in an MPC-like control scheme
- The results of the hybrid model in the multi-class case show the improvement of the prediction accuracy comparing to the pure ML and pure multi-class METANET
- The results motivate us to continue our study considering other components of the highway or studying other ML algorithms.



Thank you!